Chapter 1

AN INTRODUCTION TO 50 MORE FORMATIVE ASSESSMENT CLASSROOM TECHNIQUES (FACTS)

Classroom Snapshot of Formative Assessment in Practice

Sixth-grade students are learning to multiply and divide fractions and mixed numbers. As an exit ticket several days into the unit, they individually complete a *Slide Sort* in which they are asked to use estimation to determine whether the quotient of two numbers is less than or more than 1. The teacher collects the exit tickets looking at the students' responses and explanations prior to planning. As she looks at the students' reasoning, she realizes many are relying on rounding or calculating rather than reasoning about the size of the numbers. She realizes she needs to give them experiences with visual models using a series of problems that allow students to see patterns (ie dividing a larger fraction by a smaller fraction, dividing a smaller fraction by a larger fraction, dividing two fractions close in size, etc). She creates the following *learning intention* and *success indicators* to focus student on what they will learn and how they will show they learned it.

Learning Intention: Reasoning about the size of fractions is helpful in determining estimates for the sum of two fractions

Success Indicator #1: I can use a strategy to determine whether a quotient is larger or smaller then a given benchmark.

Success Criteria #2: I can explain the strategies I use to estimate quotients of fractions and mixed numbers.

She then plans a series of scaffolded activities that provide opportunities for students to develop a conceptual meaning of division and reason about how the sizes of the dividend and divisor affect the size of the quotient including building from whole numbers, thinking about division as how many __ are in ___? (how many ½ lbs are in 4 lbs?), and using models to find approximate and exact answers. Between activities, the teacher helps students *Take Stock* of their learning by revisiting the *Success Indicators*. During these discussions, she uses *Talk Moves* to probe further and assess students' understanding of various strategies for estimating the size of the quotient, being sure include questions for different sizes of dividends and divisors. For example, after investigating the results of dividing a smaller number by a larger number, a student claims that when dividing a smaller number by a larger number the quotient will always be less than 1. After asking students to restate and support the claim in pairs and having several pairs share with the full group, she moves in to the next activity in the lesson.

The teacher concludes the lesson by referring students to the *Learning Intention* that was posted at the start of the lesson: "Reasoning about the size of fractions is helpful in determining estimates for the sum of two fractions" She uses the *Thumbs Up, Down, and Sideways* technique—a self-assessment for students to indicate the extent to which they feel they've met the three *Success Indicators* listed at the beginning of the lesson—as evidence of meeting the *Learning Intention* and then has the students to do a quick write on *Success Indicator* 2 by asking students to explain at least two strategies for estimating the quotient of two fractions.

This brief classroom snapshot is an example of the inextricable link between formative assessment, good instruction, and learning. Formative assessment is frequently referred to as assessment *for* learning, rather than assessment *of* learning, which is summative assessment. The preposition makes a difference as formative assessment's primary purpose is to inform instructional decisions and simultaneously support learning through continuous feedback to the learner. However, a third preposition can also be added: assessment *as* learning. You can see from the snapshot provided that purposeful formative assessment classroom techniques (FACTs) become learning opportunities.

The FACTs described in this snapshot are just a few of the ways teachers can utilize various strategies to elicit students' ideas, monitor changes in their thinking, provide feedback, engage students in self-monitoring, and support reflection on learning. Throughout the process, the teacher is taking into account how well students are moving toward a learning target and what needs to be done to bridge the gap between where students are in their understanding and where they need to be. The 50 FACTs in this book, combined with the 75 FACTs in Volume 1 (Keeley and Tobey, 2011) will help you build an extensive repertoire of strategies that will inform instruction and promote learning—through a process called formative assessment. While you may be tempted to skip ahead and go directly to Chapter 3 to choose FACTs you can use in your classroom, you are encouraged to read the rest of this chapter and Chapter 2, so you can make effective use of the FACTs and strengthen your knowledge of formative assessment in mathematics instruction.

Why 50 More FACTS?

Formative assessment is a process that informs instruction and supports learning, with instructional decisions made by the teacher or learning decisions made by the student being at the heart of the process. Dylan Wiliam describes the central idea of formative assessment as follows: "Evidence about learning is used to adjust instruction to better meet students needs—in other words, teaching is *adaptive* to the learner's needs" (2011, p. 46). This overarching idea is broken down into five key strategies (Leahy et al., 2005):

- Learning intentions and criteria for success
- Designing and facilitating productive classroom discussions, activities, and tasks that elicit evidence of learning
- Providing feedback that moves learning forward
- Activating learners as instructional resources for one another
- Activating learners as the owners of their own learning

This book includes 50 new techniques that will help teachers and students utilize these five key strategies. In addition to the 75 FACTs published in the first volume of this series (Keeley and Tobey, 2011) and several of the FACTs in the science versions (Keeley 2015, 2016) that are not repeated in the mathematics versions, teachers and teacher educators now have a total of 162 FACTs to embed throughout a cycle of instruction. Table 1.1 at the end of this chapter lists the combined collection of FACTs across all the current books in this series. A rich repertoire of FACTs helps learners to interact with assessment in a variety of ways—writing, drawing, speaking, listening, questioning, investigating, modeling, and more. Furthermore, these FACTs provide mathematics-specific examples that are often lacking in general formative assessment resources.

Misunderstandings are likely to develop as a normal part of learning mathematics. These misunderstandings can be classified as conceptual misunderstandings, overgeneralizations, preconceptions, partial conceptions, and common errors. Misconceptions are a problem in mathematics for two reasons. First, when students use them to interpret or apply them to new mathematics experiences, misconceptions interfere with their learning. Second, because students have often actively constructed their misconceptions, they are emotionally and intellectually attached to them. Even when students recognize that a misconception affects their learning, they are reluctant to let go of them (Tobey and Arline 2014). For this reason it is important for teachers to have an expansive repertoire of effective techniques, such as the ones provided in this book, for uncovering, monitoring, and providing feedback on student thinking.

Another feature of the 50 new FACTs included in this book that is important in navigating today's mathematics education landscape is the connections to mathematics standards that include mathematics content, processes, and practices. Whether your state has its own mathematics standards or whether your state adopted the Common Core, the connection between the formative assessment classroom technique (FACT) and mathematics standards is included for each FACT.

The first volume of this book, *Mathematics Formative Assessment: 75 Practical Strategies for Linking Assessment, Instruction, and Learning* (Keeley and Tobey, 2011) includes important background information on formative assessment in mathematics. You are encouraged to obtain a copy of the first book as a companion to this volume, in order to read and learn more about the following:

- Types of assessment and purposes for using formative assessment
- The research that supports formative assessment
- Classroom environments that support formative assessment
- The connection between teaching and learning
- Making the shift to a formative-assessment centered classroom
- Integrating assessment and instruction
- Metacognition
- The MAIL Cycle (Mathematics Assessment, Instruction, and Learning Cycle)
- Suggestions for selecting FACTs

- Suggestions for Implementing FACTs
- Using data from FACTs

Two purposes of formative assessment that are emphasized in this collection of 50 FACTs are elicitation of student thinking related to the learning goal and supporting productive math discourse, both of which relate directly to two of NCTM's eight mathematical practices (NCTM, 2014). Each of these purposes has special nuances in mathematics that often are not explicitly addressed in general formative assessment strategies. While there are other purposes for which the FACTs are used in this book, it is important to understand these two purposes, as these are central to assessment *for* learning in mathematics.

Elicitation FACTS

"Effective teaching involves finding the mathematics in students' comments and actions, considering what students appear to know in light of the intended learning goals and progression, and determining how to give the best response and support to students on the basis of their current understandings" (NCTM, 2014, p56). Elicitation FACTs are techniques that can be used to bring this Principle to Action to life in the classroom by supporting teachers in eliciting ideas both prior to and during the instructional cycle. Many of the elicitation FACTs in this book are designed to draw out students' existing ideas, especially those that use faulty mathematics reasoning, so that responsive action can be taken. For example, many students struggle with the concept of fractions and decimals. Often the difficulty lies in overgeneralizations from their work with whole numbers. Students learning to compare fractions often incorrectly treat the numerator and denominators, as separate whole numbers incorrectly the fraction with the larger number in the denominator as the larger fraction (ie incorrectly reason that 1/8 is larger than 1/6) . When comparing decimals, students often overgenerlize the rule used when comparing whole numbers, "a number with more digits is larger" (ie incorrectly reason that 0.235 is large than 0.43). This is why elicitation of pre-existing ideas is an important part of mathematics formative assessment.

Examples of Elicitation FACTs in this book include, but are not limited to *Claim Cards, Ranking Tasks,* and *Slide Sort.* Elicitation FACTs that target pre-existing ideas are used at the beginning of a unit, cluster of lessons, or a single lesson, as well as mid-instruction, to provide an opportunity for students to surface their initial ideas and give the teacher a sense of students' thinking prior to instruction. They are used to challenge students' existing ideas or conceptual models, uncover common errors in terminology use or interpretation of representations, and expose faulty explanations of common or familiar mathematics concepts. As a formative assessment that informs instruction, it helps the teacher gauge initial student thinking, plan for enacting or modifying the lesson to follow, gauge progress mid-stream to determine a responsive action, or choose a new lesson that better addresses where students are in their mathematical understanding. Using a FACT for elicitation promotes learning by engaging students, stimulating further thinking, and setting the stage for the activities and/or discussion that will follow.

When an elicitation FACT is selected, it should be designed so that every student can have an answer or opinion, regardless of whether they are correct. The intent of using an elicitation FACT is that every student will have an opportunity to share their thinking either verbally or through writing. The data from elicitation FACTs help the teacher to set learning goals for activities designed to address students' ideas, as well as eliminate activities that may not be necessary if students demonstrate conceptual understanding.

Early in the school year as you enact formative assessment, some students may feel uncomfortable sharing their initial ideas in a whole-class setting. This will eventually change as you work to establish a classroom culture that makes it safe to discuss and evaluate peer ideas. Your goal should be to eventually move students toward public sharing and critique of their thinking. In the meantime, you might consider using an anonymous elicitation strategy such as *Fingers Under Chin* or *Extended Sticky Bars.* One way you can anonymously share students' thinking as they are writing their responses to a FACT is to say, "As I walked around the room, I noticed several of you wrote…"; or, after you have collected students' written responses to a FACT and scanned through them, you might say, "I noticed several of you think…"; or you might list students' ideas or solutions on a chart as an initial record of the class thinking without critique at this point. These anonymous techniques provide a way for students to see that not everyone has the "right answer" and that in a mathematics community, students often have alternative explanations and ways of thinking. The goal is to work toward a common, accepted understanding by considering the ideas of others and gaining new information that can be used to construct mathematical explanations. These elicitation FACTs also show students that you, the teacher, value their ideas regardless of whether they are right or wrong. Eventually students transition toward publicly sharing their thinking as they experience a learning environment where it is safe and interesting to share different ideas and ways of thinking.

First and foremost, remember the goal of elicitation is to promote student thinking, mentally commit to an answer, and participate in discussions (either immediately after an elicitation question is posed and/or in follow-up discussions) that reveal students' existing ideas. The biggest challenge for teachers right after students respond to an elicitation FACT or during follow-up elicitation discussions, is to refrain from giving the answer and/or passing judgment on students' thinking. Let students grapple with ideas, while you guide them toward understanding and a consensus of the best ideas or solutions.

Provide an opportunity for students to talk in pairs or in small groups before facilitating a whole-class discussion following use of an elicitation FACT. Circulate and listen as students discuss their ideas and defend their arguments. When pairs or small groups are talking, the teacher's role should be as a facilitator who draws out the students' ideas without indicating right or wrong answers. Once students are told or are cued as to the best answer, their thinking and questioning may stop. You want your students to keep thinking and discover the explanation for themselves, while you are gathering evidence of conceptual understanding or misunderstanding.

Supporting Productive Mathematics Discourse

"Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class. Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29). As students talk, teachers listen. A treasure trove of formative assessment data can be mined merely by carefully listening to students talk about their ideas and justify their thinking.

However, discourse alone is not enough to promote learning and inform instruction. Mathematics discourse, or math talk must be productive math talk. Productive talk is carefully orchestrated by the teacher using discussion norms in which all students are held accountable for each other's and their own learning in a respectful, safe environment. Furthermore, students must have something interesting to talk about and they must be engaged through a variety a formats and techniques the teacher purposely uses to facilitate discussion. Several of the FACTs in this book are designed to be used in various talk formats.

For example, after using an elicitation FACT in a small-group discussion, the teacher may invite the whole class to take part in a discussion to share and critique the various ideas held by students. As you chart or make a visual record of the class ideas, make sure all students have an opportunity to share any thinking that may differ from the ideas already listed, so you have a complete record of the class thinking. If you noticed during pair or small-group talk any significant thinking that was not shared during the process of listing the class ideas, you might say something like, "I heard one group talking about…" and add that to the list. Using *Talk Moves* helps encourage discussion.

Once you have generated a list of class ideas, facilitate a discussion to critique ideas by engaging the students in argumentation. The goal of argumentation is to seek understanding by putting forth ideas and persuading others to agree with your thinking, supporting it with evidence and sound mathematical reasoning. Encourage students to support their arguments in favor of or in rebuttal of one of the ideas listed by sharing evidence from previous experience, class activities, use of manipulatives, data from mathematical investigations, information from valid sources, and logic. As they engage in discussion and critique of arguments, the class usually comes to a consensus that some ideas can be discarded, narrowing the list to "our best thinking so far." This also provides the teacher with insight into student thinking and the extent to which the class is moving toward understanding the mathematics concept.

Sometimes the discussion helps the class come to an accepted common understanding of the best answer and of why it is considered the best answer. Other times, students may leave class with unresolved ideas, "hanging out in uncertainty" until the next class period, when the teacher provides an opportunity for them to re-examine their thinking, test ideas, or gather more information to resolve the differences in their thinking. Some FACTs, especially ones that involve probing questions, can be discussed and resolved in less than 45 minutes; others may take a few days of carefully designed lessons. The formative nature of the FACT allows the teacher to encourage discourse and evidence-based learning experiences that will help students reshape and refine their thinking, keeping them engaged in moving toward the learning target until they have acquired the evidence they need to develop a conceptual understanding. Examples of FACTs that are used in talk formats include, but are not limited to, *I Think-We Think, Four Corners Jigsaw, Lines of Agreement,* and *VDR.*

The challenge for teachers in using talk formats for formative assessment is deciding what to do with the student ideas you uncover during the use of talk and argument FACTs. Listening to students provides a lot of formative assessment data. Once you have uncovered these ideas, you will need to decide which ideas need to be considered in planning your next moves in instruction. It helps to categorize the ideas into valid ideas, partially valid ideas, minor misunderstandings, and major misconceptions. As you distill students' ideas that emerge from mathematics talk, think about your next steps in moving them toward the learning target. You may need to do the following:

1. Elicit further evidence of preconceptions or other conceptual obstacles
2. Confront students with examples that will challenge their existing ideas
3. Address misunderstandings related to the language of mathematics and other technical vocabulary
4. Reteach something that was taught previously but in a different context
5. Provide scaffolds to support students' developing understandings
6. Identify students who can be resources for other students during the lessons that will follow
7. Revisit the learning intentions and success indicators and adjust accordingly, depending on where students are in their understanding
8. Provide opportunities for students to access new information they can use to refine, reshape, or further develop their ideas

Several excellent resources are available to help you learn more about productive mathematics talk and how to orchestrate mathematics discussions in your classroom. The Appendix contains resources, tools, and strategies for engaging students in productive talk. These will help you use the FACTs that involve students in sharing and defending their ideas in a variety of talk formats.

Next Steps

Remember, learning is like crossing a bridge. The elicitation question surfaces where students are at the beginning of the bridge. The bridge is the connection between students' initial ideas and the mathematics understanding. The instructional opportunities you provide through talk as well as investigative experiences and problem solving will take them over the bridge, sometimes in leaps, sometimes in small steps. Eventually you want students to end up on the other side of the bridge, without being carried over the bridge by the teacher, although some students may need a helping hand to guide them. It is that moment when the student realizes he or she has crossed over the bridge on their own, leaving ideas that no longer work for him or her behind, that results in powerful conceptual learning! As you use the FACTs described in Chapter 3, think about how they can be used to help you construct that conceptual bridge. Examine the links to the mathematics practices provided in the next chapter, as many of you will be building new bridges between where your students are and where they need to be in order to address the mathematical practices expectations in the *CCSS* or your existing state standards.

As you try out the FACTs in Chapter 3, add them to your growing repertoire of purposeful formative assessment techniques. Table 1.1 combines all the FACTs across the four books that currently exist for a total of 164 techniques across mathematics and science or 125 mathematics techniques. Some of them are new, and several have been around for decades. Some are specific to mathematics, while others can be used across disciplines. Be purposeful in selecting a FACT—know why you should use it and what you will learn from using it. But start small! As Dylan Wiliam, a leader in formative assessment, wisely points out when being presented with so many formative assessment techniques and choices, “too much choice can be paralyzing—and dangerous. When teachers try to change more than two or three things about their teaching at the same time, the typical result is that their teaching will deteriorate and they go back to doing what they did before” (2011, p. 161). His advice is to choose one or two to try out in your classroom. If they seem to be effective, practice using them until they become second nature. If they do not seem to be effective, consider how you can modify them to fit your teaching style and the needs of your learners or ask other teachers who are using them for advice on how to make them work. Not all of these combined 125 techniques in mathematics will work for all teachers, but I am confident that you will find a few that will truly work for you and transform your teaching and your students’ learning.

Table 1.1 162 Science and Mathematics Formative Assessment Classroom Techniques (FACTs)

[Note to editor: Please format similar to the chart on pages 10-12 in *Science Formative Assessment- Vol. 2* (Keeley 2015)]

1. A&D Statements -
2. Always, Sometimes, or Never → |
3. Agreement Circles ¤→
4. Annotated Student Drawings ¤
5. B-D-A Drawings |
6. Card Sorts ¤→
7. Claim/Conjecture Cards | v
8. CCC—Collaborative Clued Corrections ¤→
9. Chain Notes ¤
10. C-E-O-SE |
11. Comments Only/ Comment Coding → | v
12. Commit and Toss ¤→
13. Comparison Overlap Probes |
14. Concept Attainment Cards → |
15. Concept Card Mapping ¤→
16. Concept Cartoons ¤→
17. Concept Mix-Up Probes | v
18. Confidence Level Assessment | v
19. Conjecture Cards v
20. Cover-Up v
21. Create the Problem →
22. Cross-Cutter Cards |
23. CSI (Color–Symbol–Image) |
24. Data Match ¤
25. Diagnostic Collections |
26. Directed Paraphrasing ¤
27. Enhanced Multiple Choice | v
28. Error Analysis v
29. Every Graph Tells a Story →
30. Everyday Mystery Stories |
31. Example, Non-Example → |

32. Explanation Analysis ¤
33. Extended Sticky Bars ǀ v
34. Eye Contact Partners ǀ v
35. Fact-First Questioning ¤→
36. Familiar Phenomenon Probes ¤
37. Feedback Check-ins v
38. Feedback Focused Class Discourse v
39. Feedback Sandwich v
40. Feed Up, Feedback, and Feed Forward → ǀ
41. Find Someone Who v
42. Fingers Under Chin ǀ v
43. First Word–Last Word ¤
44. Fishbowl Think Aloud ¤
45. Fist to Five ¤→
46. Flip the Question v
47. Focused Listing ¤
48. Four Corners ¤→
49. Four Corners Jigsaw ǀ v
50. Frayer Model ¤→
51. Friendly Talk Probes ¤→
52. Gallery Walk ǀ v
53. Give Me Five ¤→
54. Group Frayer Model ǀ v
55. Group Talk Feedback ǀ v
56. Guided Reciprocal Peer Questioning ¤
57. Homework Card Sort ǀ v
58. Hot Seat Questioning → ǀ
59. Human Scatterplots ¤→
60. I Think–I Rethink ǀ v
61. I Think–We Think ¤
62. I Used to Think... But Now I Know... ¤→

63. Informal Student Interviews ¤

64. Interactive Card Sorting v

65. Interest Scale ¤

66. Is It Fair? →

67. Juicy Questions ¤

68. Justified List ¤→

69. Justified True or False Statements ¤→

70. K-W-L Variations ¤→

71. Learning Goals Inventory (LGI) ¤→

72. Learning Intentions ǀ v

73. Learning Intentions Reflection v

74. Let's Keep Thinking ǀ v

75. Lines of Agreement ǀ v

76. Look Back ¤→

77. Magnetic Statements ǀ

78. Matching Cards → ǀ

79. Mathematician's Ideas Comparison →

80. Missed Conception ¤

81. More A–More B Probes → ǀ

82. Most-Least Sure About v

83. Muddiest Point ¤→

84. No-Hands Questioning ¤→

85. Now Ask Me a Question v

86. Odd One Out ¤→

87. Opposing Views Probes ¤→ ǀ

88. Overgeneralization Probes →

89. Paint the Picture ¤

90. Partner Strategy Rounds v

91. Partner Speaks ¤→

92. Pass the Question ¤

93. Pass the Problem →

94. P-E-O Probes ¤→

95. Peer-to-Peer Focused Feedback →

96. A Picture Tells a Thousand Words ¤→

97. Picture This ❙

98. Plus–Delta ❙ v

99. PMI (Plus–Minus–Interesting) ❙ v

100. POMS—Points of Most Significance ¤→

101. Popsicle Stick Questioning ¤→

102. Prefacing Explanations ¤

103. PVF—Paired Verbal Fluency ¤→

104. Question Generating ¤→

105. Questioning Cue Cards v

106. Ranking Tasks ❙ v

107. RAQ Feedback ❙ v

108. Recognizing Exceptions ¤

109. Reflect-Aloud v

110 Reflect then Assess v

111. Refutations ¤

112. Representation Analysis ¤

113. RERUN ¤

114. Response Cards →

115. Rules that Expire Probes v

116. Same A–Same B Probes →

117. Scientists' Ideas Comparison ¤

118. Seeing Structure v

119. Sequencing ¤→

120. Slide Sort ❙ v

121. Sort Envelopes v

122. Sticky Bars ¤→

123. STIP—Scientific Terminology Inventory Probe

124. Strategy Harvest →

125. Strategy Probe →

126. Structures for Taking Action v

127. Student Evaluation of Learning Gains ¤→

128. Student Interviews →

129. Success Indicators ❙ v

130: Success Indicators Problem Generating v

131. Synectics ¤

132. Take Stock v

133. Talk Moves ❙ v

134. TAR (Target, Analogy, Reflection) ❙

135. Terminology Inventory Probe ¤→

136. Ten-Two ¤→

137. Think-Alouds →

138. Thinking Log ¤→

139. Think-Pair-Share ¤→

140. Thermometer Feedback v

141. Thought Experiments ¤→

142. Three-Minute Pause ¤→

143. Three-Two-One ¤→

144. Thumbs Up, Down, and Sideways ❙

145. Traffic Light Cards ¤→

146. Traffic Light Cups ¤→

147. Traffic Light Dots ¤→

148. Traffic Light Sliders ❙ v

149. Two-Minute Paper ¤→

150. Two or Three Before Me ¤→

151. Two Stars and a Wish ¤→

152. Two-Thirds Testing ¤→

153. Vernacular Probes ❙

154. Volleyball—Not Ping Pong! ¤→

155. VDR (Vote, Discuss, Revote) ❙ v

156. Wait Time Variations ¤→

157. What Are You Doing and Why? ¤→

158. What Did I Learn Today? ǀ

159. What Stuck with You Today? v

160. Whiteboarding ¤→

161. Word Sort → ǀ

162. X Marks the Spot v

¤ In Science Formative Assessment, Volume 1: 75 Practical Strategies for Linking Assessment, Instruction, and Learning (Keeley 2008, 2015).

→ In Mathematics Formative Assessment: 75 Practical Strategies for Linking Assessment, Instruction, and Learning (Keeley and Tobey, 2011).

ǀ In Science Formative Assessment, Volume 2: 50 More Practical Strategies for Linking Assessment, Instruction, and Learning. (Keeley, 2015)

v In this Volume, Mathematics Formative Assessment: 50 More Practical Strategies for Linking Assessment, Instruction, and Learning