Introduction

A Brief History of the Common Core

Contrary to popular belief, academic standards are not new. In fact they have been around for more than 25 years. The first set of curriculum specific standards, *The Curriculum and Evaluation Standards for School Mathematics*, was released by the National Council of Teachers of Mathematics in 1989, followed by an updated set of standards, *Principles and Standards for School Mathematics*, in 2000. Both of these documents provided a vision for K–12 mathematics by grade-level band. They also formed the foundation for most states' grade-level standards.

In April 2009 the National Governor's Association and the Council of Chief State School Officers met to discuss the creation of the Common Core State Standards Initiative. The purpose of such an initiative was to develop a set of common standards across states in order to balance the quality of mathematics instruction and learning. Following that meeting, the process of writing the Common Core Standards began. The Standards Development team, led by William McCallum, Phil Daro, and Jason Zimba, included mathematicians, mathematics educators, mathematics education researchers, and classroom teachers. The process included an open invitation for feedback, not only from mathematics educators and associations, including the National Council of Teachers of Mathematics, but also from the general public. This feedback was considered and much of it was incorporated into the final document released in June 2010. Following the release of the Standards, individual states went through their own processes for reviewing, adopting, and, if necessary, ratifying the adoption of the Common Core State Standards.

The Common Core State Standards for Mathematics

"The Common Core State Standards are a clear set of shared goals and expectations for the knowledge and skills students need in English language arts and mathematics at each grade level so they can be prepared to succeed in college, career, and life" (www.corestandards.org/about-the-standards/frequently-asked-questions/#faq-2303).

The Common Core State Standards for Mathematics (CCSSM) include two critical components of learning mathematics. The Content Standards explicitly outline the mathematics we want students to know and be able to do at each grade level. The Content Standards of the Common Core are fewer in number than most previous state standards. At the same time, the expectation is that students will develop deeper understanding of that content so less time is spent on reteaching from year to year. Additionally, the Standards were carefully constructed to show connections among ideas at a grade level as well as vertical progressions across grades. For example, you will find that the Standards in Grade 3 develop from the mathematical work that students have completed in previous grades. Similarly, the Standards in Grade 4 develop from work completed in grades K–3. Thus it is important for teachers to be knowledgeable of the Standards not only at the level they are teaching but also at the preceding grade level and the following grade level.

The second group of Standards, the Standards for Mathematical Practice, describes the habits of mind that students should develop as they do mathematics. These eight Standards are the same across all grade levels, K–12. As teachers plan mathematics lessons, they should consider how students will use the Practices in learning and doing mathematics.

The Common Core Standards *are not* a curriculum. Decisions about mathematics programs, textbooks and materials, sequencing topics and units, and instructional frameworks are left for local districts to make. They do not tell teachers how to teach. It is important to remember the Standards describe what students need to know and be able to do. Schools and teachers know best how to help students reach both the Content and the Practice Standards.

The Common Core Standards *do not* dictate specific assessments. Some states will be using assessments developed by PARCC (Partnership for Assessment of Readiness for College and Careers) or SBAC (Smarter Balanced Assessment Consortium). Others will develop and use their own assessments. Other facts and information can be found at http://www.corestandards.org.

Instructional Shifts

While the Standards do not call for a particular instructional model or philosophy, they are based on the best of existing standards. What is different is that they call for specific instructional shifts: *focus*, *coherence*, and *rigor*.

Focus: The Content Standards call for greater focus on fewer topics. An examination of the mathematics standards of high-performing countries indicate that fewer, more focused topics at a grade level allow students to deepen their understanding of the mathematics and gain a stronger foundation for ongoing study of mathematics. Within the Standards, the major mathematical work of each grade level has been identified (www.corestandards.org). That means that not all of the content within a grade is emphasized equally among the Content Standards. The list of Content Standards for a grade is not linear, nor is it a checklist. Some clusters require greater emphasis than others. They take more time for students to master with depth of understanding. The major work of grades 3–5 includes multiplication and division of whole numbers and foundational understanding of fractions, leading to work in all operations with fractional numbers. This includes developing concepts, skills, and problem solving. This means the majority of instructional time in grades 3–5 (65% to 85%) should be spent on these mathematical topics. This does not mean that other Standards should be skipped. Rather, the supporting Standards should be taught to connect mathematical ideas among the essential Standards. The additional Standards provide students with experiences that will be foundational to work in future grades. Neglecting material will leave gaps in student skill and understanding (see the tables on pages 264–266).

Coherence: Many of us learned mathematics as a set of disconnected topics, with much of our skill based on tricks ("Ours is not to reason why, just invert and multiply!") or mnemonic devices ("Please Excuse My Dear Aunt Sally"). In reality, mathematics is a coherent body of knowledge made up of topics that are connected and build on each other. The call for coherence in the Content Standards ensures that there are carefully constructed progressions from grade to grade so students build new understandings on the foundations built in previous years. Each Standard is not a new topic, but an extension of previous learning. In addition to the progressions across grade levels, the Standards incorporate specific connections within a grade level. For example, as students develop conceptual understanding of multiplication and division, the relationship of these operations to each other is consistently reinforced through building conceptual understanding, procedural skills, and applying these understanding and skills to various contexts.

Rigor: The final instructional shift, rigor, refers to how we support students in developing deep understanding of each Standard. Understanding does not develop by assigning more worksheets or more difficult examples and problems. Rather, it calls for instructional practice that balances conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts.

The following descriptions of each component of rigor come from www.corestandards.org.

> *Conceptual understanding:* The Standards call for conceptual understanding of key concepts such as multiplication and division. Students must be able to access concepts from a number of perspectives in order to see mathematics as more than a set of rules or procedures.
>
> *Procedural skills and fluency:* The Standards call for speed and accuracy in calculation. Students must practice core skills, such as basic facts and multiplication and division computation, in order to have access to more complex concepts and procedures. Fluency is built upon conceptual understanding and, with elementary children, through the development of ideas through representations using concrete materials, pictures, numbers, and words.
>
> *Application:* The Standards call for students to use mathematics in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

Major Work of Grades 3–5

To help drive the focus of the Standards, at least 65% and as much as 85% of instructional time should focus on the major work for each grade level. Areas of major work include:

Grade 3: Represent and solve problems involving multiplication and division; understand properties of multiplication and the relationship between multiplication and division; multiply and divide within 100; solve problems involving the four operations, and identify and explain patterns in arithmetic; develop understanding of fractions as numbers; solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects; geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Grade 4: Use the four operations with whole numbers to solve problems; generalize place value understanding for multi-digit whole numbers; use place value understanding and properties of operations to perform multi-digit arithmetic; extend understanding of fraction equivalence and ordering; build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers; understand decimal notation for fractions, and compare decimal fractions.

Grade 5: Understand the place value system; perform operations with multi-digit whole numbers and with decimals to hundredths; use equivalent fractions as a strategy to add and subtract fractions; apply and extend previous understandings of multiplication and division to multiply and divide fractions; geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Additional information on the focus for each grade level can be found in the Resources at the end of this book.

Common Core Word Wall

The language of the Common Core differs from traditional standards. Familiarity with section names and their functions will help you to make the best use of the Common Core Standards.

Standards define what students should understand and be able to do.

Clusters summarize groups of related Standards. Note that Standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related Standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Cluster

Standard

Source: Common Core State Standards for Mathematics (www.corestandards.org)

As districts develop units of study for a grade level, careful consideration should be given to the order and connection among topics and Standards. For example, as third graders develop an understanding of the meaning of multiplication (3.OA.A.1), they use area models for multiplication to solve problems (3.OA.A.3), extend this understanding to multiply by multiples of 10 (3.NBT.A.3), and use area models to explore finding area of rectangular figures (3.MD.C.5,6,7).

The Common Core Standards for Mathematical Practice

The Common Core Standards for Mathematical Practice describe eight habits of mind teachers must incorporate into classroom instruction to help students develop depth of understanding of critical mathematical concepts. The mathematical Practices are not intended to be taught in isolation but should be integrated into daily lessons. Some lessons may focus on developing one or two of these Standards, and others may incorporate seven or all eight Standards. Note that you do not "teach" these Standards. Rather, they are the type of mathematical thinking and doing that we want students to practice as they are developing mathematical understanding.

Throughout the following chapters, we have included examples of mathematical practice that can be used in each cluster. These are not meant to limit lessons to using only those Practices. They are examples of key practices that should be included in lessons around that particular cluster. It is likely that you will use all of the Practices throughout the cluster and domain.

These eight Practices, briefly explained on the following page, are essential for student success. If students are actively engaged in using the eight Practices, they are learning rigorous, meaningful mathematics.

SFMP 1. Make sense of problems and persevere in solving them.

Students work to understand the information given in a problem and the question that is asked. They plan a solution path by choosing a strategy they can use to find a solution, and check to make sure their answer makes sense. As students in grades 3–5 work to make sense out of multiplication and division of whole numbers and fractions in these grades, using materials to solve problems helps them to develop conceptual understanding that leads to procedural fluency.

SFMP 2. Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They develop operational sense by associating contexts to numbers (thinking about 4×7 in a context of 4 baskets with 7 eggs in each basket) and associate mathematical meaning to given contexts (having 4 baskets of 7 eggs can be expressed as 4×7). Modeling problem situations with concrete materials will help students to understand the meaning of multiplication and division and build a foundation for work with fractions.

SFMP 3. Construct viable arguments and critique the reasoning of others.

Students in grades 3–5 should have many opportunities to explain their thinking and justify and communicate their conclusions both orally and in writing. Listening to others and finding how their strategies are similar may take prompting questions from the teacher such as "Why do you think that works?" or "How is your method the same as . . . ?" Mathematical discussions should be a common expectation in mathematics lessons. It will take time for students to become comfortable explaining their thinking, but this will develop over time. Explaining one's thinking helps to develop deeper conceptual understanding.

SFMP 4. Model with mathematics.

Students use various representations, models, and symbols to connect conceptual understanding to skills and applications. As students work with the big ideas of grades 3–5, they should represent mathematics situations using objects, pictures, numbers, and words. Problem solving strategies such as draw a picture, make a list, find a pattern, and write an equation have explicit connections to representations and models and can be developed at the same time.

SFMP 5. Use appropriate tools strategically.

Students consider the available tools when solving a mathematics problem. Representations such as making equal groups, arrays, and area models will help students to see the connections between multiplication and division as well as the importance of place value in understanding these operations. Bar models, area models, and the number line will help students to understand fraction number concepts. A variety of concrete materials such as cubes, tiles, straws and rubber bands, fraction bars, and physical number lines will support students in these representations.

SFMP 6. Attend to precision.

Students communicate precisely with others. Students in grades 3–5 explain their thinking using appropriate mathematical vocabulary. Students expand their knowledge of mathematical symbols that should explicitly connect to vocabulary development.

SFMP 7. Look for and make use of structure.

Students look closely to find patterns and structure in their mathematics work. For example, students begin their work with fractions using unit fractions, which helps them to better understand the meaning of the numerator and the denominator. They extend their understanding of unit fractions to other common fractions as they develop a sense of equivalence and addition and subtraction of all fractions including mixed numbers. The relationship between multiplication and division of whole numbers extends to work with fractions.

SFMP 8. Look for and express regularity in repeated reasoning.

Students notice when calculations are repeated and begin to make generalizations. By recognizing what happens when multiplying or dividing tens or hundreds, students extend that understanding to more difficult problems. Although this Standard mentions shortcuts, it should be noted that shortcuts are only appropriate when students discover them through making generalizations and understand why they work.

Effective Teaching Practices

Quality mathematics teaching is a critical key for student success. In *Principles to Actions* (2014), the National Council of Teachers of Mathematics outlines eight valuable teaching practices every teacher should incorporate to guarantee student achievement. These eight research-informed practices briefly explained below provide a foundation for effective common core mathematics teaching and student learning.

1. Establish mathematics goals to focus learning.

Establishing learning goals sets the stage and helps to guide instructional decisions. Teachers must keep in mind what is to be learned, why the goal is important, where students need to go (the trajectory), as well as how learning can be extended. Students must clearly understand the purpose of each lesson beyond simply repeating the Standard.

2. Implement tasks that promote reasoning and problem solving.

Implementing tasks that promote reasoning and problem solving provides opportunities for students to engage in exploration and encourages students to use procedures in ways that are connected to conceptual understanding. The tasks teachers choose should be built on current student understandings and have various entry points with multiple ways for the problems to be solved.

3. Use and connect mathematical representations.

Using and connecting representations leads students to deeper understanding. Different representations, including concrete models, pictures, words, and numbers, should be introduced, discussed, and connected to support students in explaining their thinking and reasoning.

4. Facilitate meaningful mathematical discourse.

Facilitating meaningful student mathematical conversations provides students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. Talking and sharing aloud can advance the mathematical thinking of the whole class.

5. Pose purposeful questions.

Posing purposeful questions reveals students' current understanding of a concept and encourages students to explain, elaborate, and clarify thinking. Asking good questions makes the learning of mathematics more visible and accessible for student examination.

6. Build procedural fluency from conceptual understanding.

Building procedural fluency from conceptual understanding based on experiences with concrete representations allows students to flexibly choose from a variety of methods to solve problems.

7. Support productive struggle in learning mathematics.

Supporting productive struggle in learning mathematics is significant and essential to learning mathematics with understanding. Productive struggle allows students to grapple with ideas and relationships. Giving young students ample time to work with and make sense out of new ideas is critical to their learning with understanding.

8. Elicit and use evidence of student thinking.

Eliciting and using evidence of student thinking helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson. Formative assessment through student written and oral ideas are excellent artifacts to assess student thinking and understanding.

How to Use This Book

The purpose of this book is to help teachers more deeply understand the mathematical meaning of each cluster and Standard within the five domains of grades 3–5. We want this book to be your toolkit for teaching the mathematics Standards, and we have left ample space for you to take notes and add ideas and other resources you have found to be helpful.

You will find each part of this book includes one domain and begins with an overview of how the domain progresses across third, fourth, and fifth grades. A list of helpful materials, reproducibles, and key vocabulary from the domain is included in the overview as well.

We track each domain across third, fourth, and fifth grades with a page for each cluster and the Standards within that cluster. A description of the cluster and how the Standards for Mathematical Practice can be incorporated into your teaching of the cluster concepts follows. Because the Standards are intentionally designed to connect within and across domains and grade levels, a list of related Standards is included in the cluster overview. We suggest that as you prepare work on a cluster you look at these Standards to have a better idea of the mathematics students learned in previous grades and where they are going in future grades. A list of all of the Standards is found in the Quick Reference Guide at the beginning of the book.

Each Standard within a cluster is explained with an example of *What the TEACHER does* to work with that Standard in the classroom followed by a description of *What the STUDENTS do*. It is important to note that most Standards will take several days, and you should be connecting conceptual understanding across Standards and domains as you teach for understanding.

Addressing student misconceptions and common errors in developing student understanding of a concept concludes the contents for each Standard.

Each cluster ends with a template for planning instruction for that cluster. At the end of each domain you will find a sample planning page based on one Standard for that domain. Also included are planning page templates for each cluster within the domain for you to duplicate and use in your planning.

In the Resources section, you will find tables that are fundamental to the Operations and Algebraic Thinking and Number and Operations in Base Ten domains. You will also find reproducibles for key materials. These are designed to be samples, and we encourage you to use them or redesign them to best meet the needs of your students. A list of our favorite resource books and high-quality online resources that are particularly useful to developing mathematical ideas in grades 3–5 are also included.

We believe that this can become your common core bible! Read it and mark it with questions, comments, and ideas. We hope that it will help you to use these Standards and good teaching practice to lay the essential foundation that will ensure your students success not only in your grade, but in all of their future study of mathematics.

Reflection Questions

1. How are the three instructional shifts called for by the Common Core similar to your current instructional practice? What is conceptual understanding? How is it different from procedural skills? What do you need to consider to teach for conceptual understanding? How can you connect conceptual understanding to help students develop procedural skills? How does the information in Table 1, page 254, on problem situations support the development of conceptual understanding?

2. The Standards for Mathematical Practice describe the habits of mind that students need for thinking about and doing mathematics. While not every Standard will be in every lesson, select one Standard at your grade level and consider some ways you can incorporate these Practices in a lesson for that Standard. How will these Practices provide you with information about student understanding? How will this help you to better assess students? How will this information help you in planning lessons?

3. The Effective Teaching Practices describe specific actions that teachers must consider in planning and implementing lessons and assessing student performance. How are these Practices connected? Work with colleagues to plan a lesson that employs all of these Practices. What needs to be considered as you consider goals for the lesson? How can you modify a traditional task so that it promotes reasoning and problem solving? What representations will help students more deeply understand the concept? What questions will you ask students? How will you connect the conceptual understanding to build procedural fluency? What questions will support students who are working to make sense of a new idea? What kind of information will you look for to help inform you instruction? (For more information on the Effective Teaching Practices, go to www.nctm.org.)