# *The Common Core Companion* at a Glance

## Operations and Algebraic Thinking

### Domain Overview

**GRADE 3**
The major work of this domain in Grade 3 is to develop students' conceptual understanding of multiplication and division by using concrete materials to model multiplication and then relate their understanding of multiplication to division. Multiplication problem situations provide a context for understanding multiplication as finding the total number of items given a number of equal groups and the number of items in each group. Division problem situations develop the meaning of division and how it is related to multiplication. When you know the total number of items and the number of groups, you can determine how many items in a group, or, when you know the total number of items and the number of items in a group, you can find the number of groups. All of these activities culminate in the expectation that students will demonstrate fluency with multiplication and division within 100 using single-digit factors.

**GRADE 4**
Students in Grade 4 continue to solve problems using the four operations with whole numbers. New to this grade level are problem situations that involve multiplicative comparisons. Students become familiar with factors and multiples and how they relate to prime and composite numbers. They work in a variety of contexts to generate and analyze patterns.

**GRADE 5**
In preparation for the Expressions and Equations domain in grades 6–8, fifth graders begin to explore, interpret, and evaluate numerical expressions. Work with patterns that began in Grade 4 extends to generating patterns, forming ordered pairs, graphing on a coordinate plane, and then analyzing the graphical representations.

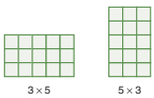
**Domain Overview:** Gives a brief description of the big ideas, allowing you to see how the mathematical ideas develop across grade levels.

**Suggested Materials for This Domain:** Provides teachers with a list of materials that will be helpful in introducing the concepts in this domain. "Reproducible" indicates that there is a handout that you can use to make multiple copies in the Resources section in the back of this book.

### SUGGESTED MATERIALS FOR THIS DOMAIN

| 3 | 4 | 5 | |
|---|---|---|---|
| ✓ | ✓ | ✓ | Hundreds chart (Reproducible 1) |
| ✓ | ✓ | ✓ | Chips, counters |
| ✓ | ✓ | ✓ | Cups, containers, other objects to represent "groups" |
| ✓ | ✓ | ✓ | Place value chart to hundreds (Reproducible 2) |
| ✓ | ✓ | ✓ | Square tiles |
| ✓ | ✓ | ✓ | Grid paper (Reproducible 3) |
| ✓ | ✓ | ✓ | Pattern blocks |
| ✓ | ✓ | ✓ | Number cards (such as a deck of playing cards) |

### KEY VOCABULARY

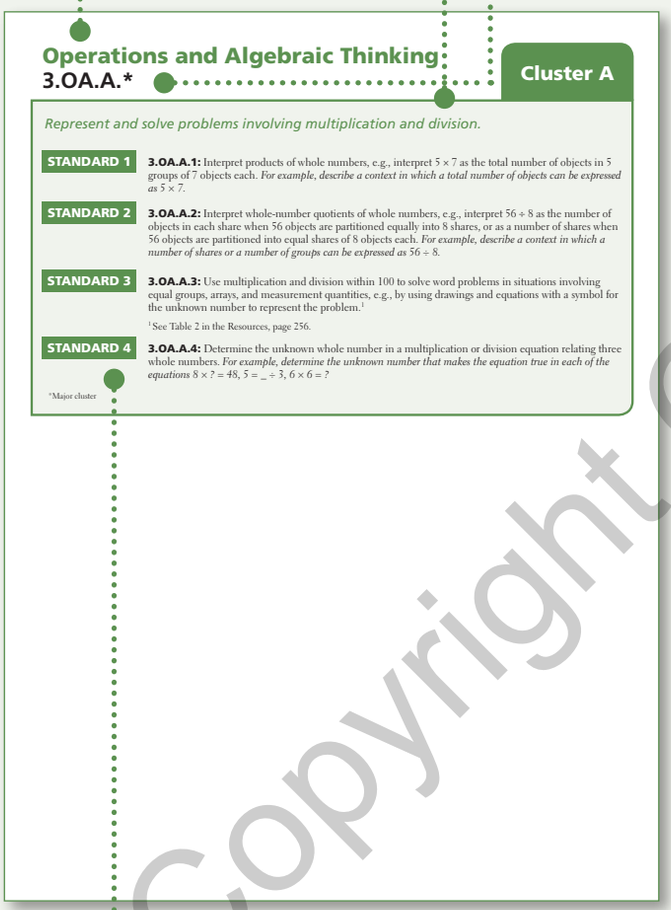
| 3 | 4 | 5 | |
|---|---|---|---|
| ✓ | ✓ | ✓ | **add** to combine or join together<br>related words: *add, and, plus, join, put together, (+)* |
| ✓ | ✓ | ✓ | **addend** any of the numbers added to find a sum |
| ✓ | ✓ | ✓ | **area model** a concrete model for multiplication or division made up of a rectangle. The length and width represent the factors and the area represents the product.<br>  <br>3 × 5 5 × 3 |
| ✓ | ✓ | ✓ | **array model** a concrete model for multiplication in which items are arranged in rows and columns. Each row (or column) represents the number of groups and each column (or row) represents the number of items in a group.<br>  <br>5 × 4 = 20 5 rows of 4 = 20 |
| ✓ | ✓ | ✓ | *** associative property of multiplication** an extension of the commutative property; to change the order and group two factors to find convenient products (such as 10) in order to make the multiplication easier. Students may begin to use parentheses at this level.<br>7 × 8 × 5 = 7 × (8 × 5) = 7 × 40 = 280 |
| ✓ | ✓ | ✓ | *** commutative property of multiplication** reversing the order of the factors does not change the product<br>8 × 5 = 40 and 5 × 8 = 40 therefore the product of 8 × 5 = 5 × 8 |

**Key Vocabulary:** Vocabulary included in the domain with grade levels at which that term is used. This terminology can be used for building a word wall in the classroom. Students should be able to use these terms in talking about mathematics in discussions unless otherwise noted. Standard for Mathematical Practice 6: Attend to Precision calls for students to use mathematical terminology appropriately.

**Domain:** General mathematical topic for this group of Standards.

**Cluster:** Statements that summarize groups of related Standards. Note that Standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Identifying number for this cluster:** Grade, domain, cluster



**Operations and Algebraic Thinking**
**3.OA.A.***

**Cluster A**

*Represent and solve problems involving multiplication and division.*

**STANDARD 1** **3.OA.A.1:** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5 × 7.

**STANDARD 2** **3.OA.A.2:** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* 56 ÷ 8.

**STANDARD 3** **3.OA.A.3:** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Table 2 in the Resources, page 256.

**STANDARD 4** **3.OA.A.4:** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

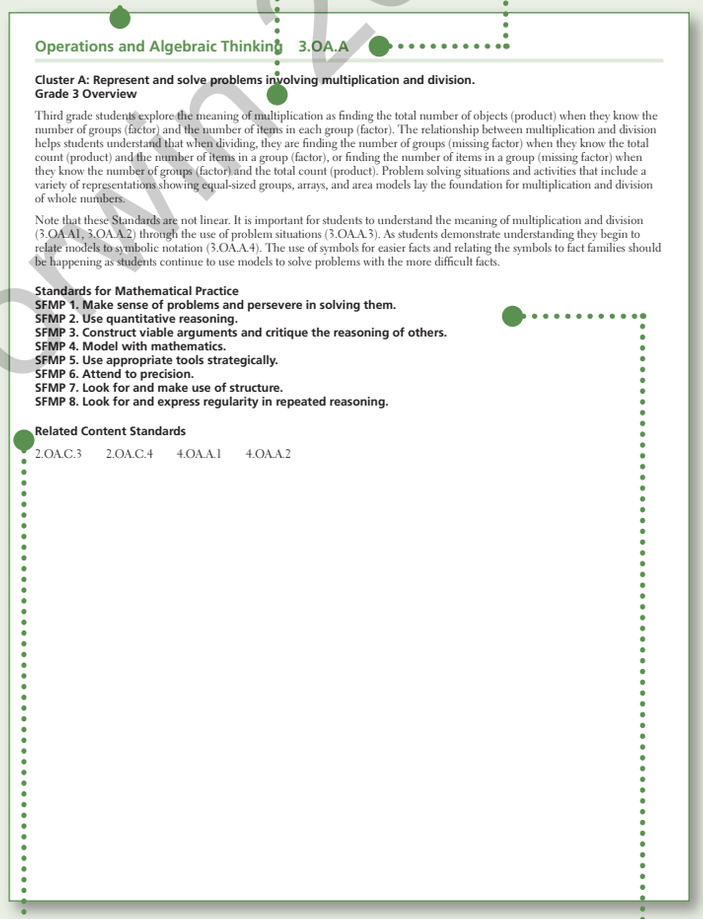
*Major cluster

**Standards:** Mathematical statements that define what students should understand and be able to do.

Grade, domain, cluster

Each cluster begins with a brief description of the mathematics in that cluster.

Domain



**Operations and Algebraic Thinking 3.OA.A**

**Cluster A: Represent and solve problems involving multiplication and division.**
**Grade 3 Overview**

Third grade students explore the meaning of multiplication as finding the total number of objects (product) when they know the number of groups (factor) and the number of items in each group (factor). The relationship between multiplication and division helps students understand that when dividing, they are finding the number of groups (missing factor) when they know the total count (product) and the number of items in a group (factor), or finding the number of items in a group (missing factor) when they know the number of groups (factor) and the total count (product). Problem solving situations and activities that include a variety of representations showing equal-sized groups, arrays, and area models lay the foundation for multiplication and division of whole numbers.

Note that these Standards are not linear. It is important for students to understand the meaning of multiplication and division (3.OA.A1, 3.OA.A.2) through the use of problem situations (3.OA.A.3). As students demonstrate understanding they begin to relate models to symbolic notation (3.OA.A.4). The use of symbols for easier facts and relating the symbols to fact families should be happening as students continue to use models to solve problems with the more difficult facts.

**Standards for Mathematical Practice**
**SFMP 1. Make sense of problems and persevere in solving them.**
**SFMP 2. Use quantitative reasoning.**
**SFMP 3. Construct viable arguments and critique the reasoning of others.**
**SFMP 4. Model with mathematics.**
**SFMP 5. Use appropriate tools strategically.**
**SFMP 6. Attend to precision.**
**SFMP 7. Look for and make use of structure.**
**SFMP 8. Look for and express regularity in repeated reasoning.**

**Related Content Standards**
2.OA.C.3 2.OA.C.4 4.OA.A.1 4.OA.A.2

**Related Content Standards:** Provides a list of Standards connected to this topic in other grade levels as well as Standards in this grade level related to this topic that are in other domains. We recommend you look at the related Standards as you plan your instruction for this cluster.

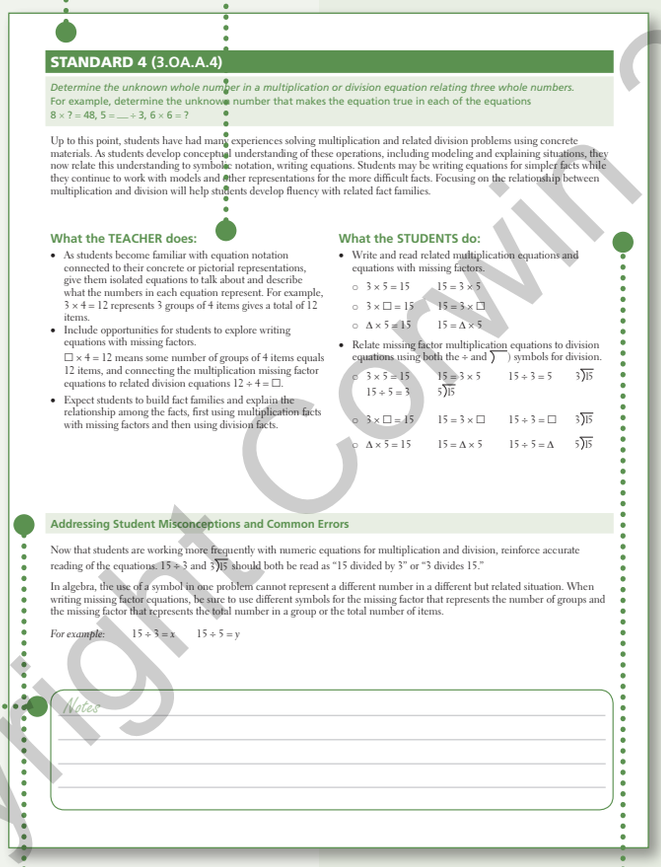
**Standards for Mathematical Practice:** Although it is likely you will use a variety of Standards for Mathematical Practice in teaching each cluster, this section gives examples of how you might incorporate some of the practices into your instruction on this topic.

You will find the following components for each Standard in the cluster:

**Standard:** The Standard as written in the Common Core followed by an explanation of the meaning of the mathematics in that Standard, including examples.

**Notes:** We have included space beneath each Standard for you to take notes as you study its mathematical content. This might include vocabulary, materials, resources you want to use, or an explanation of the Standard in your own words.

**What the TEACHER does:** An overview of actions the teacher might take in introducing and teaching the Standard. This is not meant to be all-inclusive, but rather to give you an idea of what classroom instruction might look like. We included illustrations of how to use materials to teach a concept when using models and representations is called for in the Standard.



**STANDARD 4 (3.OA.A.4)**

*Determine the unknown whole number in a multiplication or division equation relating three whole numbers.*
For example, determine the unknown number that makes the equation true in each of the equations
$8 \times ? = 48$, $5 = \_\_ \div 3$, $6 \times 6 = ?$

Up to this point, students have had many experiences solving multiplication and related division problems using concrete materials. As students develop conceptual understanding of these operations, including modeling and explaining situations, they now relate this understanding to symbolic notation, writing equations. Students may be writing equations for simpler facts while they continue to work with models and other representations for the more difficult facts. Focusing on the relationship between multiplication and division will help students develop fluency with related fact families.

**What the TEACHER does:**

- As students become familiar with equation notation connected to their concrete or pictorial representations, give them isolated equations to talk about and describe what the numbers in each equation represent. For example, $3 \times 4 = 12$ represents 3 groups of 4 items gives a total of 12 items.
- Include opportunities for students to explore writing equations with missing factors.
  $\square \times 4 = 12$ means some number of groups of 4 items equals 12 items, and connecting the multiplication missing factor equations to related division equations $12 \div 4 = \square$.
- Expect students to build fact families and explain the relationship among the facts, first using multiplication facts with missing factors and then using division facts.

**What the STUDENTS do:**

- Write and read related multiplication equations and equations with missing factors.
  - $3 \times 5 = 15$ $\quad 15 = 3 \times 5$
  - $3 \times \square = 15$ $\quad 15 = 3 \times \square$
  - $\Delta \times 5 = 15$ $\quad 15 = \Delta \times 5$
- Relate missing factor multiplication equations to division equations using both the ÷ and $)\overline{\quad}$ symbols for division.
  - $3 \times 5 = 15$ $\quad 15 = 3 \times 5$ $\quad 15 \div 3 = 5$ $\quad 3)\overline{15}$
    $15 \div 5 = 3$ $\quad 5)\overline{15}$
  - $3 \times \square = 15$ $\quad 15 = 3 \times \square$ $\quad 15 \div 3 = \square$ $\quad 3)\overline{15}$
  - $\Delta \times 5 = 15$ $\quad 15 = \Delta \times 5$ $\quad 15 \div 5 = \Delta$ $\quad 5)\overline{15}$

**Addressing Student Misconceptions and Common Errors**

Now that students are working more frequently with numeric equations for multiplication and division, reinforce accurate reading of the equations. $15 \div 3$ and $3)\overline{15}$ should both be read as "15 divided by 3" or "3 divides 15."

In algebra, the use of a symbol in one problem cannot represent a different number in a different but related situation. When writing missing factor equations, be sure to use different symbols for the missing factor that represents the number of groups and the missing factor that represents the total number in a group or the total number of items.

*For example:* $15 \div 3 = x$ $\quad 15 \div 5 = y$

*Notes*

**Addressing Student Misconceptions and Common Errors:** Each Standard concludes with a description of possible student misconceptions or common student errors around the Standard and suggested actions to address those misconceptions or errors.

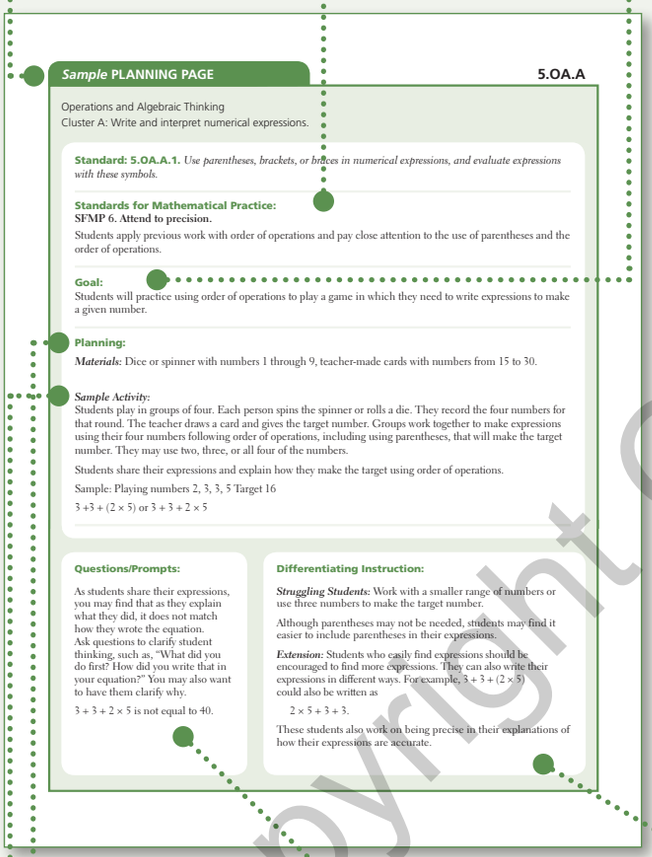
**What the STUDENTS do:** Some examples of what the students might be doing as they explore and begin to understand the Standard. Again, this is not intended to be directive but rather to frame what student actions might look like.

**Sample Planning Page:** We have provided one complete sample planning page for one Standard at the end of each cluster in each grade level. Although it may not be the final lesson plan, it does provide the areas you should consider while planning your lessons.

**Goal:** What is the purpose of this activity and how does it connect to previous (and future) ideas?

**Standards for Mathematical Practice:** What mathematical practices can you emphasize in this activity?

**Planning Page:** A planning template is provided at the end of each cluster. This template is provided for your use as you consider instructional actions around a particular Standard. You might want to make copies of this page and use them for each Standard within the cluster. This is not intended to be an all-inclusive lesson plan. Rather, it gives you a place to record your thoughts about teaching a mathematical topic as you read the Standard.



*Sample* PLANNING PAGE 5.OA.A

Operations and Algebraic Thinking
Cluster A: Write and interpret numerical expressions.

**Standard: 5.OA.A.1.** *Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.*

**Standards for Mathematical Practice:**
**SFMP 6. Attend to precision.**
Students apply previous work with order of operations and pay close attention to the use of parentheses and the order of operations.

**Goal:**
Students will practice using order of operations to play a game in which they need to write expressions to make a given number.

**Planning:**
***Materials:*** Dice or spinner with numbers 1 through 9, teacher-made cards with numbers from 15 to 30.

***Sample Activity:***
Students play in groups of four. Each person spins the spinner or rolls a die. They record the four numbers for that round. The teacher draws a card and gives the target number. Groups work together to make expressions using their four numbers following order of operations, including using parentheses, that will make the target number. They may use two, three, or all four of the numbers.

Students share their expressions and explain how they make the target using order of operations.

Sample: Playing numbers 2, 3, 3, 5 Target 16

$3 + 3 + (2 \times 5)$ or $3 + 3 + 2 \times 5$

**Questions/Prompts:**

As students share their expressions, you may find that as they explain what they did, it does not match how they wrote the equation. Ask questions to clarify student thinking, such as, "What did you do first? How did you write that in your equation?" You may also want to have them clarify why.

$3 + 3 + 2 \times 5$ is not equal to 40.

**Differentiating Instruction:**

***Struggling Students:*** Work with a smaller range of numbers or use three numbers to make the target number.

Although parentheses may not be needed, students may find it easier to include parentheses in their expressions.

***Extension:*** Students who easily find expressions should be encouraged to find more expressions. They can also write their expressions in different ways. For example, $3 + 3 + (2 \times 5)$ could also be written as

$2 \times 5 + 3 + 3$.

These students also work on being precise in their explanations of how their expressions are accurate.

PLANNING PAGE 5.OA.B

Operations and Algebraic Thinking
Cluster B: Analyze patterns and relationships.

**Standard:**

**Standards for Mathematical Practice:**

**Goal:**

**Planning:**

***Materials:***

***Sample Activity:***

**Questions/Prompts:**

**Differentiating Instruction:**

***Struggling Students:***

***Extension:***

**Planning:** What materials will you use to teach this Standard?

**Sample Activity:** An example of an activity that addresses this standard is provided.

**Questions/Prompts:** It is important to anticipate student thinking throughout the activity. Think about the questions or prompts you might give to help build student understanding and encourage student thinking so you do not find yourself telling students what to do.

**Differentiating Instruction:** How can you tweak the activity to address the needs of students who are struggling? How can you extend the activity for students who demonstrate understanding of the mathematics?

**Resources:** In the Resources section at the end of the book you will find an overview of each practice for teachers of grades 3–5 to consider and implement: Table 1, Addition and Subtraction Situations, Grades 3–5, which explains problem solving situations for addition and subtraction, and Table 2, Multiplication and Division Situations, Grades 3–5, which explains problem solving situations for multiplication and division and provides strategic competencies for students. Other resources include Table 3, which offers an overview of the Standards for Mathematical Practice and what each practice Standard means for students in grades 3–5; Table 4, the effective teaching practices from NCTM's *Principles to Actions;* and reproducibles for some of the materials recommended for each grade level.

An example of a problem that exemplifies the situation.

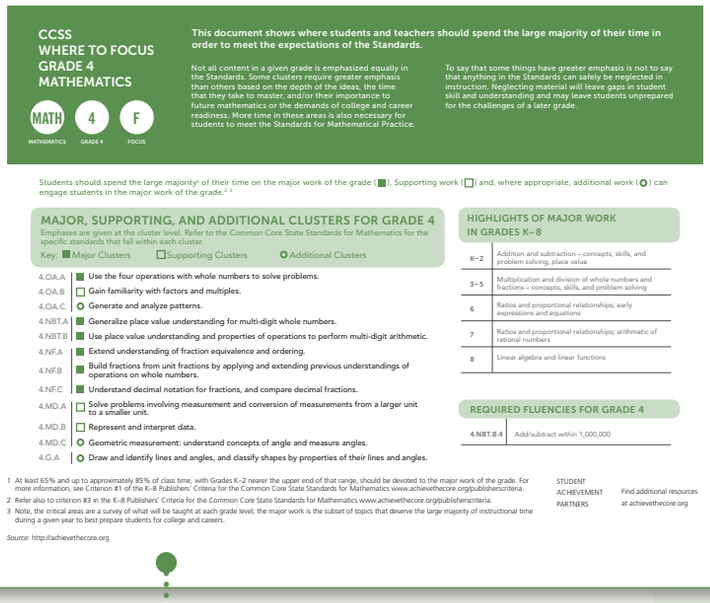
Equation(s) that represent the situation.

## Table 1 Addition and Subtraction Situations, Grades 3–5

| Situation | Problem | Equation(s) |
|---|---|---|
| **Add to—result unknown** | Frank had 235 pennies. Mark gave him 156 more. How many pennies does Frank have? | **235 + 156 = *p*** |
| **Add to—change unknown** | Frank had 235 pennies. Mark gave him some more. Now Frank has 391 pennies. How many pennies did Mark give to Frank? | **235 + *p* = 391** |
| **Add—start unknown** | Frank had some pennies in his piggy bank. Mark gave him 156 more. Now Frank has 391 pennies. How many pennies did Frank have at the beginning? | ***p* + 156 = 391** |
| **Take from—result unknown** | Frank had 45 pennies. He spent 29 pennies on a package of jawbreakers. How many pennies does he have left? | **45 – 29 = *p*** |
| **Take from—change unknown** | Frank had 45 pennies. He spent some pennies on a pack of jawbreakers. Now Frank has 16 pennies. How much did he spend on the jawbreaker? | **45 – *p* = 16** |
| **Take from—start unknown** | Frank had some pennies in his bank. He spent 29 pennies on a package of jawbreakers. Now he has 16 pennies. How many pennies did Frank have in his bank? | ***p* – 29 = 16** |
| **Put together take apart—total unknown** | Anna has been saving coins. She has 348 pennies and 267 nickels. How many coins does she have? | **348 + 267 = *p*** |
| **Put together take apart—addend unknown** | Anna 615 coins in her piggy bank. She has 348 pennies and the rest are nickels. How many coins are nickels? | **615 = 348 + *p*** |
| **Put together take apart—addends unknown** | Anna has 11 coins. Some are pennies and some are nickels. How many pennies and how many nickels could Anna have? | **1 + 10 = 11**<br>**2 + 9 = 11**<br>**3 + 8 = 11**<br>**4 + 7 = 11**<br>**5 + 6 = 11**<br>**6 + 5 = 11**<br>**7 + 4 = 11**<br>**8 + 3 = 11**<br>**9 + 2 = 11**<br>**10 + 1 = 11** |

Various problem situations for addition and subtraction.

## CCSS Where to Focus Grade 4 Mathematics



CCSS WHERE TO FOCUS GRADE 4 MATHEMATICS

MATH (MATHEMATICS) 4 (GRADE 4) F (FOCUS)

**This document shows where students and teachers should spend the large majority of their time in order to meet the expectations of the Standards.**

Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the Standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Students should spend the large majority[1] of their time on the major work of the grade (■). Supporting work (□) and, where appropriate, additional work (○) can engage students in the major work of the grade.[2,3]

**MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 4**

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters □ Supporting Clusters ○ Additional Clusters

| | | |
|---|---|---|
| 4.OA.A | ■ | Use the four operations with whole numbers to solve problems. |
| 4.OA.B | □ | Gain familiarity with factors and multiples. |
| 4.OA.C | ○ | Generate and analyze patterns. |
| 4.NBT.A | ■ | Generalize place value understanding for multi-digit whole numbers. |
| 4.NBT.B | ■ | Use place value understanding and properties of operations to perform multi-digit arithmetic. |
| 4.NF.A | ■ | Extend understanding of fraction equivalence and ordering. |
| 4.NF.B | ■ | Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. |
| 4.NF.C | ■ | Understand decimal notation for fractions, and compare decimal fractions. |
| 4.MD.A | □ | Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. |
| 4.MD.B | □ | Represent and interpret data. |
| 4.MD.C | ○ | Geometric measurement: understand concepts of angle and measure angles. |
| 4.G.A | ○ | Draw and identify lines and angles, and classify shapes by properties of their lines and angles. |

**HIGHLIGHTS OF MAJOR WORK IN GRADES K–8**

| | |
|---|---|
| K–2 | Addition and subtraction – concepts, skills, and problem solving; place value |
| 3–5 | Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving |
| 6 | Ratios and proportional relationships; early expressions and equations |
| 7 | Ratios and proportional relationships; arithmetic of rational numbers |
| 8 | Linear algebra and linear functions |

**REQUIRED FLUENCIES FOR GRADE 4**

| | |
|---|---|
| 4.NBT.B.4 | Add/subtract within 1,000,000 |

1 At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see Criterion #1 of the K-8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.

2 Refer also to criterion #3 in the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.

3 Note, the critical areas are a survey of what will be taught at each grade level; the major work is the subset of topics that deserve the large majority of instructional time during a given year to best prepare students for college and careers.

STUDENT ACHIEVEMENT PARTNERS Find additional resources at achievethecore.org

Source: http://achievethecore.org

**CCSS Where to Focus Mathematics:** The major content focus for each grade level is identified on the grade-level focus charts included in the Resources.

**Reproducibles:** A variety of reproducibles can be duplicated and used by students in the classroom when working with concrete materials.

**Reproducible 1. Hundreds Chart**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |