What Successful Math Teachers Do, Grades 6–12

To Barbara for her support, patience, and inspiration.
To my children and grandchildren, David, Lauren, Lisa, Danny, Max,
Sam, and Jack, whose futures are unbounded.
And in memory of my dear parents, Alice and Ernest,
who never lost faith in me.

—Alfred S. Posamentier

To my Mom and Dad, whose unceasing love and support, and examples of
integrity and hard work have converged to allow me a
life limited only by my imagination.

—Terri L. Germain-Williams

To my wife, Tae Jin, who has made every sacrifice to ensure
my happiness and success.
To my children, Jennifer and Rebecca, who are an eternal source of pride
and joy, and in memory of my parents, Stanley and Beatrice,
who were always there for me.

—Daniel Jaye

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Second Edition

80 Research-Based Strategies for the Common Core–Aligned Classroom

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Prologue

As a direct result of federal pressure on the states to continuously improve their instructional program and ensure that all students are being reached in the teaching process, teachers are being called on to meet professional standards and base their work on research-proven methods of teaching. Educational research, often conducted at universities or on educational sites by university researchers, is reported in educational journals and is most often read by other researchers. All too often, the style in which research reports or articles on research findings are reported is not friendly or appealing to the classroom teacher. The very community—classroom teachers—that could benefit enormously from the findings of many of these educational initiatives rarely learns about these endeavors. It is the objective of this book to bring some of the more useful research findings to the classroom teacher. In our quest for the most salient research findings, we were guided by the Common Core State Standards and the National Council of Teachers of Mathematics standards. Rather than merely presenting the research findings that support these standards, we have attempted to convert them into useful classroom strategies, thus capturing the essence of the findings and at the same time putting them into a meaningful context for the practicing mathematics teacher.

This book is to serve as a resource for mathematics teachers. It should provide a portal to access the many worthwhile findings resulting from educational, psychological, and sociological research studies done in Europe and in the United States. Heretofore, teachers have had very few proper vehicles for getting this information, short of combing through the tomes of research reports in the various disciplines. This book is designed to provide an easy way for the classroom teacher to benefit from the many ideas embedded in these academic exercises.

The book is designed to be an easy and ready reference for the mathematics teacher—both preservice and inservice. It consists of 11 chapters, each with a theme representing one aspect of the typical instructional program and each guided by the Common Core Standards. Each chapter presents a collection of teaching strategies, concisely presented in a friendly format:

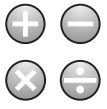
The Strategy

This is a simple and crisp statement of the teaching strategy we recommend.

What the Research Says

This offers a discussion of the research project that led to the strategy. This section should give the teacher some confidence in, and a deeper understanding of, the principle being discussed as a "teaching strategy."

Teaching to the National Council of Teachers of Mathematics Standards



Here we present the salient National Council of Teachers of Mathematics standard that we are supporting with the strategy.

Aligning to the Common Core State Standards

This section not only helps explain the standards but also shows how they can be met.

Classroom Applications

This section tells the teacher how the teaching strategy can be used in the mathematics instructional program. Where appropriate, some illustrative examples of the teaching strategy in the mathematics classroom are provided.

Precautions and Possible Pitfalls

This is the concluding section for each strategy and mentions some of the cautions that should be considered when using this teaching strategy so that the teacher can avoid common difficulties

before they occur, thereby achieving a reasonably flawless implementation of the teaching strategy.

Sources

These are provided so that the reader may refer to the complete research study to discover the process and findings in detail.

We see this book as a first step in bringing educational research findings to the practitioners: the classroom teachers. Perhaps teachers will see that there is much to be gained to enhance teaching by reviewing educational research with an eye toward implementing the findings in their instructional program. Furthermore, it would be highly desirable for researchers to make more of an effort to extend their publications/findings to the classroom teacher. To do otherwise would make the entire activity of educational research irrelevant.

As you read the many instructional suggestions offered in this book, we hope you will continuously think of yourself as the teacher who might implement them. Remember, your personality plays a large role in mapping out an overall instructional strategy. Each teacher brings to the classroom various strengths, and therefore, the research we bring to the reader should be viewed in that context. Nevertheless, the specific research-based tips and strategies offered here will help you focus on certain aspects of your teaching. Teachers who continuously self-evaluate their instructional performance will, undoubtedly, become master teachers.

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About the Authors

Alfred S. Posamentier is Dean of the School of Education and Professor of Mathematics Education at Mercy College, New York. For the previous 40 years he held these same positions at The City College of The City University of New York. He is the author and coauthor of more than 55 mathematics books for teachers, students, and the general readership. He is also a frequent commentator in newspapers on topics relating to education.

After completing his BA degree in mathematics at Hunter College of The City University of New York, he took a position as a teacher of mathematics at Theodore Roosevelt High School in the Bronx (New York), where he focused his attention on improving the students' problem-solving skills and at the same time enriching their instruction far beyond what the traditional textbooks offered. He also developed the school's first mathematics teams (both at the junior and senior levels). He is currently involved in working with mathematics teachers and supervisors, nationally and internationally, to help them maximize their effectiveness.

Immediately upon joining the faculty of The City College (after having received his master's degree there), he began to develop inservice courses for secondary school mathematics teachers, including such special areas as recreational mathematics and problem solving in mathematics.

Dr. Posamentier received his PhD from Fordham University (New York) in mathematics education and has since extended his reputation in mathematics education to Europe. He has been visiting professor at several European universities in Austria, England, Germany, and Poland, and at the University of Vienna, he was Fulbright Professor in 1990.

In 1989, he was awarded the title of Honorary Fellow at the South Bank University (London, England). In recognition of his outstanding teaching, The City College Alumni Association named him Educator of the Year in 1994, and New York City had May 1, 1994, named in his honor by the President of the New York City Council. In 1994, he was also awarded the Grand Medal of Honor by the Federal Republic of Austria. In 1999, upon approval of Parliament, the president of the Federal Republic of Austria awarded him the title of University Professor of Austria; in 2003, he was awarded the title of *Ehrenbürger* (Honorary Fellow) of the Vienna University of Technology, and he was awarded (June 2004) the Austrian Cross of Honor for Arts and Science, First Class by the President of the Federal Republic of Austria. In 2005, he was elected to the Hall of Fame of the Hunter College Alumni Association, and in 2006, he was awarded the Townsend Harris Medal from The City College of New York. Other honors bestowed upon Dr. Posamentier include Education Leader of the Year, *Education Update* newspaper, 2009; Educator of the Year, The City College of New York Education Alumni Association, 2009; New York State Mathematics Education Hall of Fame, New York State Association of Mathematics Supervisors, 2009; and the Christian-Peter Beuth Prize 2009, Beuth Society and University of Applied Science, Berlin, Germany, 2010.

He has taken on numerous important leadership positions in mathematics education locally. He was a member of the New York State Education Commissioner's Blue Ribbon Panel on the Math-A Regents Exams. He served on the Commissioner's Mathematics Standards Committee, which was charged in 2004 with rewriting the Standards for New York State, and he is on the New York City Public Schools Chancellor's Math Advisory Panel.

After 40 years on the faculty of The City College of New York, and now three years as Dean of the School of Education at Mercy College, New York, he is still a leading commentator on educational issues and continues his longtime passion of seeking ways to make mathematics interesting to teachers (see *Math Wonders: To Inspire Teachers and Students* [2003] and *The Art of Motivating Students for Mathematics* Instruction [2011]), students, and the public—as can be seen from his latest books: *Math Charmers: Tantalizing Tidbits for the Mind* (2003); *π, A Biography of the World's Most Mysterious Number* (2004); *101+ Great Ideas for Introducing Key Concepts in Mathematics, Second Edition* (2006); and *The Fabulous Fibonacci Numbers* (2006); *Problem Solving Strategies for Efficient and Elegant Solutions* (2008);

Mathematical Amazements and Surprises: Fascinating Figures and Noteworthy Numbers (2009); *The Pythagorean Theorem* (2010); *The Glorious Golden Ratio* (2012); *The Secrets of Triangles: A Mathematical Journey* (2012); and *Magnificent Mistakes in Mathematics* (2013).

Terri L. Germain-Williams is Assistant Professor of Mathematics Education at Mercy College, teaching courses in methods of teaching mathematics as well as Assessment and Evaluation. She also works with a number of schools and organizations as an educational consultant, supporting schools in the areas of mathematics instruction, scheduling and programming, the implementation of small learning communities, data and accountability, strategic planning, and leadership.

Germain-Williams graduated with a bachelor's degree in mathematics from Adelphi University and began her career as an intern at Mepham High School in Bellmore, New York, during a fifth-year master's program. Upon graduating with her master's degree and completing her internship, Germain-Williams began her teaching career as an eighth-grade mathematics teacher in Jericho, New York. The lure to make a difference in an area of high need brought Germain-Williams to join the team of founding teachers of the Bushwick School for Social Justice housed in the Bushwick High School Campus in Brooklyn, New York. This group of dedicated educators brought the vision of the planning team to reality when, within four years, tripled the historically low graduation rate and her own personal vision: to have a class of students learn calculus before enrolling in college.

Upon graduating from Queens College's School Supervision and Administration program, Germain-Williams was fortunate to join the administrative team at the Academy of Urban Planning (AUP). At AUP, Germain-Williams focused on supporting student data practices and provided professional development and support to the mathematics and science departments. During her tenure as Assistant Principal, she was accepted into the PhD in Mathematics Education Program at Teachers College, Columbia University, where she is now in the final stages.

Prior to accepting the position with Mercy College Graduate School of Education, Germain-Williams worked as an Achievement Manager with the New York City Department of Education,

supporting more than 25 K–12 schools in the areas of instruction, strategic planning, professional development, federal and state data and accountability, scheduling/programming, and student services.

Daniel Jaye is the Chief Academic Officer and Director of Academic Affairs at the Solomon Schechter Day School of Bergen County. He lectures frequently and enjoys presenting interesting techniques in problem solving as well as problems that provide enrichment for the mathematics classroom. He is also interested in comparing math standards throughout the nation and the world.

Jaye graduated from The City College of New York with a major in mathematics and began his career teaching mathematics at Seward Park High School (New York City). After one year, he was invited to teach at the prestigious Stuyvesant High School (New York City), where he distinguished himself by teaching the entire range of high school mathematics courses.

After receiving his master's degree in mathematics education from The City College of New York, Jaye took an interest in guiding student research projects in mathematics. Shortly thereafter, he served as the math research coordinator and coordinated the submission of thousands of student-generated research papers to local and national competitions, including the Westinghouse and Intel Science Talent Search Competitions. In 2001, he was awarded the Mathematical Association of America's Edyth Sliffe Award for Excellence in Teaching. He was also the recipient of Education Update's Outstanding Teacher of the Year award in 2004.

After 25 years of outstanding teaching, Jaye was selected as Assistant Principal for the Stuyvesant High School Department of Mathematics. He immediately began to put his energies into creating more opportunities for talented and gifted students to study advanced mathematics. He was chosen to be Executive Director of the New York City Math Team, where he coordinated the training activities of the 100 members of the team. In 2001, he created and directed the CCNY Summer Scholars Academy in Mathematics and Science. This program provides advanced courses in mathematics and science and is supplemented by a stellar guest lecture series featuring noted mathematicians, scientists, and educators.

In 2004, Jaye was chosen to serve on the New York State Math Standards Committee, which authored new state standards in mathematics. In 2004, he was elected President of the Association of Mathematics Assistant Principals for Supervision (New York City) and was awarded the Phi Delta Kappa Leadership in Education award. He has served on the New York City Public Schools Chancellor's Math Advisory Panel and the New York State Mathematics Curriculum Committee.

In 2006, Jaye was selected to lead the Bergen Academies in Hackensack, New Jersey, as Principal and Director, where he guided the institution to national acclaim as the sole recipient of the Intel School of Distinction Award for academic excellence. In 2010, he assumed the position of Chief Academic Officer and Director of Academic Affairs at the Solomon Schechter Day School of Bergen County.

Jaye's passion for teaching and interest in mathematics standards and problem solving were inspirational in creating this book.