Preface

Over the years, problem solving has emerged as one of the major concerns at all levels of school mathematics. In fact, the National Council of Supervisors of Mathematics (NCSM) points out that "learning to solve problems is the principal reason for studying mathematics" (NCSM, 1977, p. 1). Furthermore, the National Council of Teachers of Mathematics (NCTM) in their *Professional Standards for Teaching Mathematics* state rather boldly that "problem solving, reasoning and communication are processes that should pervade all mathematics instruction and should be modeled by teachers" (NCTM, 1991, p. 95). This is further buttressed in their *Principles and Standards for School Mathematics*, where they explicitly say that "problem solving is an integral part of all mathematics learning" (NCTM, 2000, p. 52). We are in complete agreement! In fact, we would go one step further: We feel that problem solving is not only a skill to be taught in mathematics but also a skill that will be carried over to everyday "problems" and serve a person well throughout his or her life.

In many cases, students seem to feel that a problem can only be solved in a single way, specific to the "type" of problem being taught (i.e., motion problems, age problems, mixture problems, etc.). Students often feel that an algebraic approach is the only procedure that will "work." But where does this misconception come from?

In fact, it is often the teachers who themselves are not even aware of the many problem-solving strategies that can be used to provide efficient and elegant solutions to many problems. It is often they who unconsciously convey to their students the notion that problems can only be solved using an algebraic approach. While we would agree that algebra is a most powerful tool, it is still only one of the many approaches students should be made aware of when it comes to problem solving. This book is a result of our many years of efforts to make teachers and students aware of this most important aspect of teaching mathematics. Furthermore, it is designed for the classroom teacher who has a sincere desire to help students succeed as problem solvers both in mathematics and beyond. This is not to say that the book cannot be used by students directly; quite the contrary! However, its "tone" is directed to the teacher, who "should

engage students in mathematical discourse about problem-solving—(which) includes discussing different solutions and solution-strategies for a given problem, (and) how solutions can be extended and generalized" (NCTM, 1991, p. 95).

In this book, we examine 10 strategies that are widely used in problem solving both in mathematics and real-life situations. In the mathematics classroom, these strategies provide an alternate plan for resolving many problem situations that arise within the curriculum. We have selected about 200 problems to illustrate these strategies realizing that teachers will apply these strategies to their regular instructional program. To do this, we recommend a careful review and study of the examples provided for each strategy so that the strategy eventually becomes a genuine part of the teacher's thinking processes or, one might say, a part of the teacher's arsenal of problem-solving tools.

While it is true that most of the problems can be solved using the tried-and-true techniques of algebra and geometry (and we *do* show these solutions as well), the purely "mechanical" approach often masks some of the efficiency, beauty, and elegance of mathematics. In many cases, the problem-solving strategies presented make the solution of a problem much easier, much "neater," and much more understandable and, thereby, enjoyable!

Throughout the book we try to show how each of these strategies occurs and ought to be consciously used in real-life situations. Many people already make use of these strategies without realizing it. This carry-over into life outside of the school adds importance to the mathematics our students study and will ultimately improve their everyday performances. We believe that you and your students alike can profit from a careful reading (and working along) of this book. As you examine each problem, take the time to solve it in any way you wish, or perhaps in a variety of ways. Compare your solutions to the ones provided. (Naturally, we welcome any clever alternatives to those in the book.) Most important, try to absorb the impact of the application of the problem-solving strategies and how they contribute to the beauty and power of mathematics. All the better if you can carry this motivated feeling over to your students.

Understand our feeling that problem solving must be the cornerstone of any successful mathematics program and then try to infuse this same enthusiastic feeling and attitude in your daily teaching. This concentrated effort will make you a better problem solver and in turn help your students to also become better problem solvers. Not only will their attitude toward mathematics improve but so will their skills and abilities. This is our ultimate goal.

REFERENCES

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